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NOISE IN PULSED MICROWAVE SYSTEMS

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Abstract

The relationship between the noise spectrum of a pulsed microwave signal and that of the same signal operating in the CW mode is derived theoretically and verified through measurements. It is shown that a simple closed form solution exists for wideband white noise but more involved calculations are required otherwise.

Introduction

In pulsed transmitter applications, it is often necessary to calculate the noise spectrum of a pulsed signal from that of the CW signal, and vice versa. In one example, since it is much easier to measure the CW noise spectrum than the pulsed spectrum, it would be advantageous to only measure noise in the CW mode and then be able to predict the noise performance when the system is operated in the pulsed mode. In an example of the reverse situation, if the CW equivalent noise spectrum for a transmitter, which can only operate in the pulsed mode, is needed to predict radar performance, the approach is to measure the pulsed noise spectrum and then to calculate the CW equivalent spectrum from it. In this article, the relationship between the CW and pulsed noise spectra is established through the convolution integral. Using this technique, it is shown that the pulsed noise spectrum can be calculated from the CW spectrum in a straightforward manner. On the other hand, given a pulsed spectrum, the CW equivalent can be calculated exactly only for some special cases such as when the noise is uniform. For the non-uniform cases, estimated results can still be obtained by assuming some functional dependence of the noise to offset frequency. Verification of the model is carried out through measurement of a uniform and a $1/f^n$ noise spectra under CW and pulsed conditions.

Mathematical Model

The mathematical model is developed using standard Fourier transform techniques [1]. Given two signals $s_1(t)$ and $s_2(t)$, and their corresponding Fourier transforms $S_1(\omega)$ and $S_2(\omega)$, the Fourier transform of the signal $s_3(t)$ where

$$s_3(t) = s_1(t)\, s_2(t) \tag{1}$$

is

$$S_3(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_1(x)\, S_2(\omega - x)\, dx \ . \tag{2}$$

Let $s_2(t)$ be an infinite pulse train of unit amplitude, pulse width τ and period T, $S_2(\omega)$ is the well known sampling function:

$$S_2(\omega) = 2\pi \frac{\tau}{T} \sum_{k=-\infty}^{\infty} \frac{\sin(\omega\tau/2)}{(\omega\tau/2)}\, \delta(\omega - 2\pi k/T) \tag{3}$$

where $\delta(x)$ is the delta function which is non-zero only at x=0. Substituting Eq. (3) into Eq. (2) results in:

$$S_3(\omega) = \frac{\tau}{T} \int_{-\infty}^{\infty} S_1(x) \sum_{k=-\infty}^{\infty} \frac{\sin[(\omega-x)\tau/2]}{[(\omega-x)\tau/2]}\, \delta(\omega - x - 2\pi k/T)\, dx$$

$$= \frac{\tau}{T} \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} S_1(x) \frac{\sin[(\omega-x)\tau/2]}{[(\omega-x)\tau/2]}\, \delta(\omega - x - 2\pi k/T) dx$$

$$= \frac{\tau}{T} \sum_{k=-\infty}^{\infty} S_1(\omega - 2\pi k/T) \frac{\sin(\pi k\tau/T)}{(\pi k\tau/T)} \ . \tag{4}$$

Without going into more rigorous mathematics, some observations can be made of this result. Pulsing the CW spectrum $S_1(\omega)$ causes the CW noise components at radian frequency $(\omega - 2\pi k/T)$, modified by the amplitude $(\tau/T)\sin(\pi k\tau/T)/(\pi k\tau/T)$, to add up at radian frequency ω in the pulsed spectrum. If $S_1(\omega)$ is a noise spectrum so that the component at $(\omega-2\pi k/T)$ and $(\omega-2\pi \ell/T)$ are uncorrelated for $k \neq \ell$, then the spectral lines add up in a RMS fashion, resulting in:

$$S_3(\omega)_{RMS} = \frac{\tau}{T} \sqrt{\sum_{k=-\infty}^{\infty} \left\{ S_1(\omega - 2\pi k/T) \frac{\sin(\pi k\tau/T)}{(\pi k\tau/T)} \right\}^2} \tag{5}$$

If the noise spectrum is uniform so that:

$$|S_1(\omega)| = N_o \ , \tag{6}$$

Eq. (5) becomes:

$$S_3(\omega)_{RMS} = \frac{\tau}{T} N_o \sqrt{\sum_{k=-\infty}^{\infty} \left\{ \frac{\sin(\pi k\tau/T)}{(\pi k\tau/T)} \right\}^2}$$

$$= N_o \sqrt{\frac{\tau}{T}} \tag{7}$$

in the limit $\tau << T$. Furthermore, dividing Eq. (7) by Eq. (6) yields:

$$\frac{S_3(\omega)_{RMS}}{|S_1(\omega)|} = \sqrt{\frac{\tau}{T}} \qquad (8)$$

which states that pulsing a CW white noise spectrum causes the noise level to be reduced by a factor equal to the square root of the duty factor (τ/T). Another observation that can be made is that if $S_1'(\omega)$ is the spectrum of a CW carrier with amplitude V_o and radian frequency ω_o, then:

$$S_1'(\omega) = \pi V_o [\delta(\omega-\omega_o) + \delta(\omega+\omega_o)] \quad . \qquad (9)$$

Substituting Eq. (9) into Eq. (4) results in:

$$S_3'(\omega) = \frac{\tau}{T} \sum_{k=-\infty}^{\infty} \pi V_o [\delta(\omega-\omega_o-2\pi k/T) + \delta(\omega+\omega_o-2\pi k/T)] \frac{\sin(\pi k\tau/T)}{(\pi k\tau/T)} \qquad (10)$$

which states that pulsing a CW carrier causes the single spectral line to become a series of lines separated by radian frequencies $2\pi(k/T)$ and shaped by the sampling function in amplitude. At $\omega = \omega_o$, $S_3'(\omega)$ becomes:

$$S_3'(\omega_o) = \frac{\tau}{T} \pi V_o \qquad (11)$$

and

$$\frac{S_3'(\omega_o)}{S_1'(\omega_o)} = \frac{\tau}{T} \qquad (12)$$

indicating that pulsing a carrier causes the spectral line at ω_o to be reduced by the duty factor (τ/T).

If $S_1(\omega)$ is not a constant but can be written as a function of ω, Eq. (4) can still be evaluated numerically. An example of this calculation where $S_1(\omega) \propto 1/\omega^n$ is given in the next section together with the measured data.

Measurement

The technique of measuring CW and pulsed noise associated with a microwave carrier has been well documented in the literature [2-4]. For the purpose of verifying the mathematical model which relates the CW and pulsed spectra, it is sufficient to perform a simplified set of measurements. Since the detection process is equivalent to frequency translating the carrier to zero frequency and the noise spectrum to baseband, detecting a pulsed carrier together with its noise sidebands and then filtering out the dc is equivalent to pulsing the detected spectrum at baseband without the carrier. Therefore, in the following experiments, the effect of detecting a pulsed carrier together with its noise sidebands is simulated by simply pulsing the noise spectrum at baseband. Figure 1 shows the measurement block diagram. A white noise generator of at least 5 MHz bandwidth is applied to an adjustable bandwidth filter. This noise distribution is then pulse modulated with 10 μsec wide pulses at 10 percent duty cycle, and the resulting noise sidebands are measured on a spectrum analyzer.

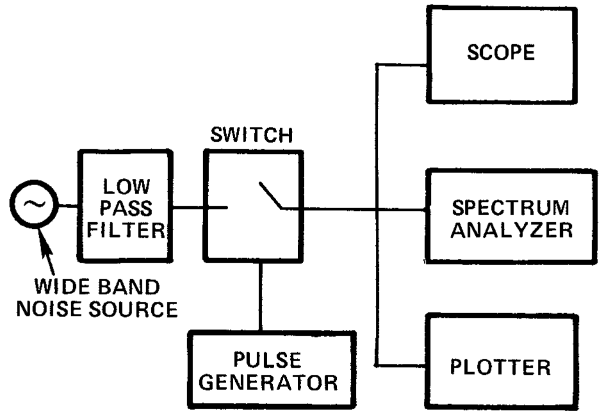

Figure 1 - Measurement Block Diagram

In the uniform noise case the filter is removed and the noise is then measured with and without pulse modulation. Comparison of the CW and pulsed noise distributions on the spectrum analyzer from dc to 200 kHz (Figure 2) revealed that the input noise was equally distributed over frequency in both cases and the amplitude for the modulated case is lower by 10 dB which is 20 log $\sqrt{\tau/T}$ in agreement with Eq. (8).

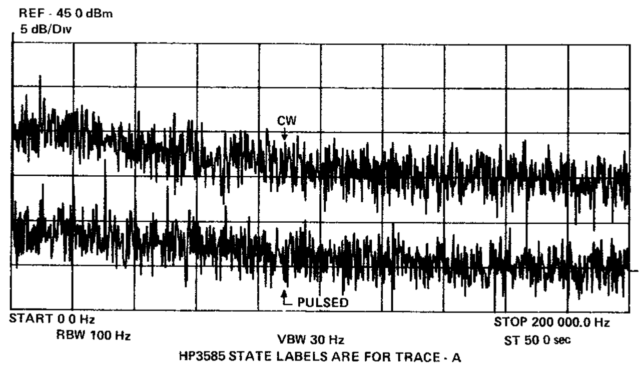

Figure 2 - Measured CW and Pulsed Uniform Noise Spectra 10 μsec Pulse Width 10 Percent Duty Cycle

For the non-uniform case a double pole RF filter with a 10 kHz cutoff is applied to the white noise generator. The spectrum analyzer then displays the shaped roll off prior to pulse modulation. After modulation the noise distribution is found to be dependent upon the CW input noise distribution as shown in Figure 3. The calculated pulsed noise result using Eq. (5) is shown in Figure 4. The result shows good agreement with the measured pulsed noise spectrum shown in Figure 3. The CW noise model which approximates the measured CW noise as shown in Figure 3 is also shown in Figure 4.

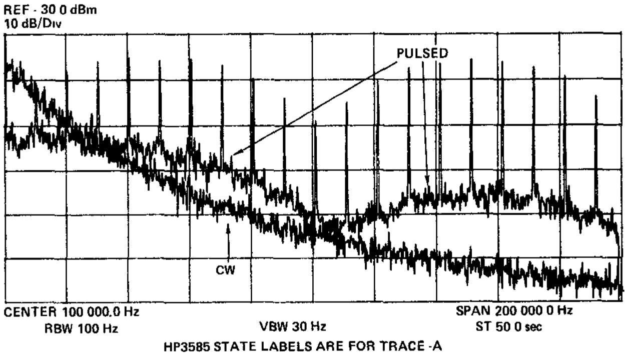

Figure 3 - Measured CW and Pulsed Non-Uniform Noise Spectra 10 μsec Pulse Width 10 Percent Duty Cycle

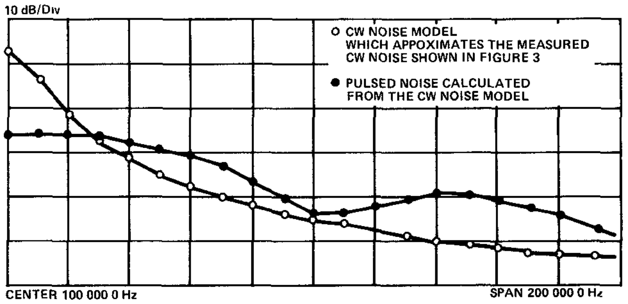

Figure 4 - Calculated Pulsed Non-Uniform Noise Spectrum (from a CW Noise Model which Approximates the Measured CW Noise Spectrum) 10 μsec Pulse Width 10 Percent Duty Cycle.

Conclusions

The relationship between pulsed noise and CW noise was established and verified through measurements. It is shown that if the noise spectrum is uniform, the relationship between the CW and pulsed noise spectra reduces to a simple algebraic equation. If the noise is not uniform, the pulse noise is related to the CW noise through a convolution integral. The integral allows the pulsed noise spectrum to be readily calculated from a given CW spectrum. In order to do the opposite,namely, to calculate the CW spectrum from the pulsed spectrum, some functional dependence of the CW noise on frequency (such as $1/f^n$) must be assumed. These results are applicable to the characterization of pulsed systems where noise performance is of concern.

References

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